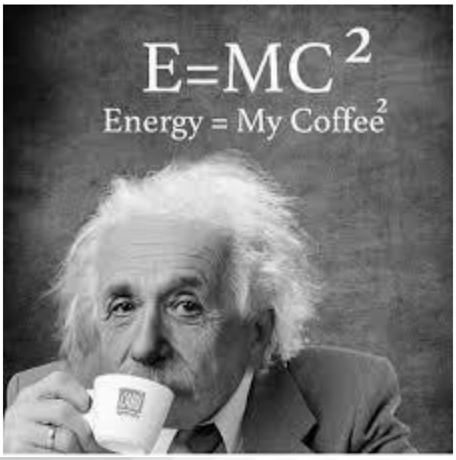
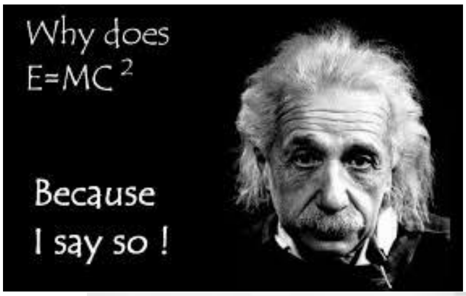
$E=mc^2$





"Now that desk looks better. Everything's squared away, yessir, squaaaaaared away."


E=ma²
E=mb²
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ESCALATION EQUALS MORE CASUALTIES, too.
E=mc²
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Deriving the equation -- Advanced

This page takes a formal approach to deriving Einstein's E = mc^2 using calculus and assumes a background knowledge of basic physics, as well as relativity as related on the other pages in this series. The equation is also derived without calculus on this page, and the terms used in the equation are explained here.

We start by noting that energy is the integral of force with respect to distance, so kinetic energy K can be defined by:

$$K = \int_0^s F ds$$

Where F is the force in the direction of displacement ds and s is the distance over which the force acts.

Using Newton's second law of motion force F can be shown as:

$$F = \frac{d(mv)}{dt}$$

Thus the equation for kinetic energy K can now be shown as:

$$K = \int_0^s \frac{d(mv)}{dt} ds = \int_0^{mv} vd\,(mv) = \int_0^v vd\left[\frac{m_0 v}{\sqrt{1-v^2/c^2}}\right]$$

Note that the velocity limit is c (the speed of light). At c time dilation becomes 100% and distances in the direction of motion shrink to zero, hence a body at this speed will not experience time or distance and so its velocity is set as the upper limit.

We now integrate by parts :

$$\int xdy = xy - \int ydx$$

To yield:

$$K = \frac{m_0 v^2}{\sqrt{1-v^2/c^2}} - m_0 \int_0^v \frac{vdv}{\sqrt{1-v^2/c^2}}$$

$$= \frac{m_0 v^2}{\sqrt{1-v^2/c^2}} + \left[m_0 c^2 \frac{m_0 v}{\sqrt{1-v^2/c^2}}\right]_0^v$$

$$= \frac{m_0 c^2}{\sqrt{1-v^2/c^2}} - m_0 c^2$$

$$= mc^2 - m_0 c^2$$

The result shows that the kinetic energy of a body is equal to the increase in its mass as a consequence of its relative motion multiplied by c^2. This can be rearranged to show:

$$mc^2 = m_0 c^2 + K$$

If the kinetic energy is decreased so that K = 0 the body will be stationary, but will still possess energy m_0c^2. In other words the body contains energy E_0 when stationary relative to its frame and will have mass m_0. This is called the rest mass. This is shown as:

$$E = E_0 + K$$

where:

$$E_0 = m_0 c^2$$

This, then, completes the derivation of E = mc^2 for a body at rest. For a moving body its total energy is given by:

$$E = mc^2 = \frac{m_0 c^2}{\sqrt{1-v^2/c^2}}$$